Process Capability Assessment When Multiple Characteristics Exist: A Composite Approach Using Six Sigma Metrics

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Abstract

Process capability assessment constitutes the most important part of assessment of process performance and historically occupies a prominent place in the area of statistical process control. Process capability measures popularly known as the "indices" are commonly used to assess the processes and the required benchmarking and comparisons are done. These studies usually involve a single quality characteristic and hence do not pose any issues. However it is well known that any process or product comprises multiple characteristics and hence an array of measures corresponding to each of the quality characteristic will emerge. This makes it difficult to judge the overall quality of the product or the process in the absence of one single comprehensive measure. Several researchers have come out with alternative ways of assessing the process capability to deal with multiple characteristics. Considering the popularity of the Six Sigma approach towards quality improvement, this paper examines the various situations *and explores the applicability of six sigma metrics to similar situations. A comprehensive approach using a combination of Six Sigma metrics along with a comparison of process capability indices is illustrated. The analysis is carried out with examples drawn from actual cases.*

Keywords: *Process, Capability, Six Sigma, Indices, Metrics, Multiple, Characteristics*

Introduction

Process capability assessment is an important step before a process is declared suitable for full scale production. Several measures were developed to express the process capability and those measures were also tested for their ability to reflect the process quality in a simple way, (Juran, 1974, Bothe, 1992). While Zabecki (1986) describes the capability ratio, Kane (1986), Chan, et al, (1988) and Spiring, (1991) describe the process capability indices. A lesser known measure namely Percent Tolerance Consumed by Capability (PTCC) can be seen in Hradesky (1988).

Process capability indices are the most widely used measures of process capability and are aptly described as “unitless measures” that help in process evaluation and comparison, (Kane, 1986). These indices help to express the process potential and performance in a simple way, usually as a single number. Further, they can be translated to determine the process percent rejects, which is the most vital information regarding a process. Other prominent researchers who have examined the process capability measures include Clements (1989) who has examined the case of non-normal distribution while assessing process capability and Barnett (1990) who has explored the sample based analysis ad corresponding confidence intervals for the process capability indices. Boyles (1991) has used the Taguchi loss function concept to build the process capability measure. Rodriguez (1992) has provided an overview of developments in process capability analysis. In addition, Raouf and Ali (1992) present a procedure to find an optimal ongoing process capability index. Deleyd (1998) has investigated the gap between the theory and practice of process capability studies. However, all these indices have one major limitation, that they address the issue considering only one quality characteristic at a time. For instance, if a product has a dozen characteristics, then as many indices as there are characteristics will have to be calculated and analyzed, in order to draw a meaningful conclusion. However, decision making becomes very difficult when an array of measures is involved. This acts as a serious disadvantage, since it is well known that any product or component in reality will have a multi dimensional or multicharacteristic features and consequently gives rise to a cluster of capability indices. In order to simplify the task of decision making in such situations, it becomes necessary to develop a suitable multicharacteristic measure which can take into account more than one characteristic at a time. Further, if this multicharacteristic measure can be linked to the conceived product quality and process output, then, it would be easier for documentation and communication and eventually for initiating appropriate decisions quickly. Because of this reason a number of researchers have worked in the area of multivariate quality control and assessment of process quality when there are more than one process or more than one product characteristic being formed concurrently or sequentially.

Review of Literature for Multivariate Quality Control

Multivariate quality control has attracted a lot of attention for a long time, as seen in the literature. In a very early paper Ghare and Togerson (1968) have explored the possibility of plotting control charts when several quality characteristics exist. Chiatello (1974) has examined how process control can be improved through multivariate analysis. Chua and Montgomery (1991, 1992) report that significant work has been done in this area with the primary concern of controlling the quality when several parameters (variables) are involved pertaining to a product or service. They have come out with a quality control scheme for multivariate quality control. Patel (1973) has analyzed the process quality control methods for non-normal distributions like binomial and Poisson distribution under multivariate quality control.

Several researchers have attempted to develop process capability measures for an entire product or a series of processes that result in a manufactured product. Notable contributors are Chen, et. al. (2001, 2004) who have extended the concepts to entire product families. Similarly Nien (2001) has illustrated how process capability indices from a series of processes can be combined to form an overall measure of process quality. Huang and Chen (2003) have taken up the case of process capability assessment when multiple characteristics exist with bilateral specifications.

As stated by Yang and Trewn (2004) the word multivariate not only means many variables, but also means that these variables might be correlated. Statistical methods involve a range of techniques and procedures for analyzing, interpreting, and displaying data, and making decisions based on data usually involving one variable. Extending the approach, multivariate statistical methods are collection of methods and procedures that analyze, interpret, display, and characterize data pertaining to several variables correlated or not.

In the recent times the assessment of quality when multiple characteristics are present has been rigorously explored by several authors wo have all suggested different methods to arrive at an overall measure of process capability. These researchers include Sabriari and Abdollahzadeh (2009), Wang (2010), Goethals and Cho (2011), and Pearn, et. al (2011,2013), who have all suggested the ways of handling the multivariate quality control situation.

While most of the multivariate quality control schemes have dealt with control chart procedures for more than two variables, several researchers have attempted to establish a measure of process capability to deal with several variables at a time. Chan et al.(1991) have proposed a multivariate measure of process capability that considers proximity to the target value, and the dispersion around the target.

Hubele et al. (1991) have proposed a three component capability vector, based on a bivariate probability contour. However they state that an interpretation of an exact value of this component is not recommended. Besides, the practical utility of the proposed measure needs to be as certained.

It can be observed that, multivariate quality control is definitely more complicated than univariate quality control. While the situations involving only two variables can be effectively handled, more than two variables tend to create more difficulties during analysis and interpretation. Further, the statistical aspects like, distribution of individual variables and joint distribution of variables, have to be closely examined and hence, call for considerable statistical skill and knowledge. This is perhaps one reason why multivariate process capability measures have not been developed and implemented in the same scale as univariate measures. Consequently, the current procedure continues to be applying the univariate measures for each of the quality characteristics in a multicharacteristic situation. This method can be of limited use because of the following reasons: For every measure one calculation has to be made and makes it time consuming. Particularly when the number of characteristics increases sharply, the situation becomes very complex and unmanageable. Further the benchmark values may be satisfied in some cases but may not satisfy in others. This again leads to inconclusiveness and overall assessment becomes tricky. All these observations strongly indicate that there is a need to develop useful and simple methods to deal with multicharacteristic situations. It is to be noted here that multicharacteristic situations are a statistician's delight but practitioners will not be able to understand or apply these measures in day to day situations because of the complexity of calculations and difficulty in apprehending the situation. Hence simpler and perhaps a commonly used measure needs to be developed. In this paper, such an attempt has been made using the six-sigma approach but not completely abandoning the process capability indices.

Need for A Comprehensive Approach

With the advent of globalization, it has become apparent that companies need to think on a global scale and be competitive in the domestic as well as the international market. It is in this context that any quality measurement should be conveyed to all the partners and stakeholders of the company. Obviously a metric commonly accepted across the world would prove to be convenient and effective in demonstrating the quality standards maintained by the company. Secondly, the company will also be assessed against the global benchmarks and thus would be able to provide an overall picture of the company's relative position on the quality scale. In this regard, Six Sigma metrics which are now commonly used by both manufacturing and non-manufacturing companies enables such comparisons on the global scale. For a long time one commonly used measure of quality was the percent rejects coming out of a process. But, it describes only one aspect of a process, namely, percent of products not conforming to specifications. However, the real economic interpretation lies in describing the variability among the products and the closeness of the quality characteristics to the target value. In this paper, first a problem pertaining to a real life case is illustrated. The limitations of the existing measures are brought out and the need for a multicharacteristic measure is justified. Then a review of available methods and approaches is made and their applicability to practical cases is discussed. The details of a new measure proposed are presented and the same real life case is analyzed again using the proposed measure. The merits and weaknesses of the new measure are discussed to draw relevant conclusions

Case Study of A Threaded Fastener

In this case study a threaded fastener used in automobile sub-assemblies is considered and the data is taken from an old paper which has reported a multicharacteristic process capability assessment using Taguchi loss function, (Jagadeesh and Babu, 1995). The component in this case namely the threaded fastener is checked for acceptance based on three dimensions and the corresponding specifications are shown in Table 1.

Table 1 : Quality characteristics of the component

Specification limits	Total Length	Thread Length	Core Diameter
Lower Specification Limit (LSL)	24.36 mm	15.00 mm	3.33 mm
Upper Specification Limit (USL)	25.00 mm	No USL	3.53 mm

Using appropriate measuring instruments the three quality characteristics namely the total length, the thread length, and the core diameter are measured and recorded for 125 components classified into 25 samples of 5 components each. These data values are shown in Table 2. The data values indicate the three quality characteristics namely total length, thread length, and core diameter measured in the same order for all the 125 components with a total of 375 dimensional values.

Using these values the different process capability indices and other relevant values are calculated and shown in Table 3. All the values, are in millimeters (mm) and restricted to four decimal places. Further, using these process capability values relevant comments are made about the process as shown in Table 4.

Table 2 : Data values of the 125 components under three catagories with sample size of 5.

S. No.	Total Length					Thread Length					Core Diameter				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	24.5	25.0	24.6	24.5	24.7	15.0	14.5	15.0	15.5	15.0	3.45	3.40	3.39	3.41	3.40
2	24.7	24.6	24.8	24.5	24.9	15.5	15.0	15.0	15.0	15.5	3.42	3.41	3.46	3.41	3.40
3	24.4	24.9	24.6	24.9	24.9	15.0	15.0	15.5	15.0	15.5	3.38	3.42	3.43	3.43	3.41
4	24.5	24.8	24.4	24.5	24.6	15.0	15.0	15.5	14.5	15.0	3.38	3.42	3.41	3.41	3.38
5	24.6	24.8	24.8	24.5	24.5	14.5	15.0	15.0	15.0	15.0	3.41	3.41	3.39	3.43	3.42
6	24.8	24.5	24.6	24.5	24.7	15.0	15.0	15.0	15.0	15.0	3.40	3.44	3.41	3.42	3.43
7	24.8	24.7	24.6	24.7	24.6	15.0	15.0	15.5	15.0	15.0	3.40	3.43	3.40	3.39	3.38
8	24.8	24.7	24.6	24.8	24.6	15.0	15.0	15.5	15.0	15.5	3.44	3.42	3.38	3.43	3.43

9	24.8	24.7	24.6	24.8	24.7	15.0	15.0	15.5	15.0	15.0	3.38	3.40	3.42	3.43	3.38
10	24.7	24.4	24.5	24.6	24.5	14.5	15.0	15.0	15.0	15.0	3.42	3.38	3.43	3.43	3.40
11	24.7	24.5	24.6	24.6	24.8	15.0	15.0	15.0	15.0	15.0	3.40	3.42	3.37	3.41	3.41
12	24.5	24.8	24.6	24.6	24.5	15.0	15.5	15.0	15.0	14.5	3.43	3.42	3.43	3.42	3.41
13	24.4	24.7	24.5	24.6	24.6	15.0	15.0	15.0	15.0	15.5	3.38	3.43	3.41	3.38	3.38
14	24.5	24.6	24.7	24.4	24.5	15.0	14.5	15.0	15.0	15.0	3.40	3.38	3.41	3.35	3.40
15	24.6	24.8	24.6	24.8	24.8	15.0	15.5	15.0	15.0	14.5	3.38	3.44	3.42	3.41	3.41
16	24.3	24.5	24.6	24.7	24.7	15.0	15.0	15.0	15.5	15.0	3.42	3.39	3.42	3.41	3.44
17	24.5	24.8	24.7	24.4	24.6	14.5	15.0	15.0	15.0	15.5	3.38	3.37	3.42	3.37	3.40
18	24.6	24.6	24.5	24.5	24.6	15.0	15.0	15.0	15.0	15.0	3.42	3.40	3.40	3.39	3.41
19	24.7	24.5	24.6	24.7	24.6	15.0	15.0	15.5	15.0	15.0	3.45	3.40	3.38	3.42	3.38

20	24.6	24.8	24.6	24.6	24.7	15.0	14.5	15.0	15.0	15.0	3.43	3.38	3.39	3.43	3.40
21	24.5	24.5	24.7	24.7	24.7	15.0	15.5	15.0	15.5	15.0	3.37	3.45	3.35	3.42	3.38
22	24.6	24.8	24.6	24.7	24.7	15.5	15.0	15.0	15.5	14.5	3.43	3.43	3.43	3.37	3.38
23	24.8	24.8	24.8	24.7	24.7	15.0	15.0	15.5	15.0	15.5	3.38	3.40	3.39	3.43	3.38
24	24.7	24.5	24.5	24.6	24.7	15.5	15.0	15.5	15.0	15.0	3.43	3.39	3.42	3.47	3.39
25	24.6	24.6	24.7	24.7	24.8	15.0	15.0	15.0	15.0	15.5	3.40	3.41	3.40	3.39	3.42

Table 3. Process capability calculations

Process parameters	**Total Length**	**Thread Length**	**Core Diameter**
Lower specification limit, LSL	24.4	15.0	3.35
Upper specification limit, USL	25.4	16.0	3.45
Target value, T	24.9	15.5	3.40
Process Mean, ì	24.6384	15.0600	3.4069
Process standard deviation, ó	0.1300	0.2587	0.0226
Process capability indices			
Process capability, 6 ó	0.7800	1.5523	0.1355
Process potential index, Cp	1.2821	0.6442	0.7378
Process performance index, Cpk	0.6113	0.0773	0.6362
Taguchi capability index,			
Cpm	0.5705	0.3265	0.7058

Table 4. Comments based on the process capability assessment of the three quality characteristics

Basis of comments	**Total length**	**Thread length**	**Core diameter**
Mean	Significantly away from the target	Significantly away from the target	Close to the target
Standard deviation	Slightly more variability	Large amount of variability	Less variability
Process capability	Process spread is less than tolerance spread. Marginally adequate	Process spread wider than the tolerance spread. Defects bound to occur.	Process spread is wider than the tolerance spread. Defects bound to occur.
Cp	Capable process	Incapable process	Incapable process
Cpk	The process is currently meeting the specifications	The process is not able to produce within the tolerance	The process is currently not able to meet the specifications
Cpm	Since less than 1.0, indicates mean away from the target and also more variable	Highly insufficient value indicating poor centering and wider variability	Since less than 1.0, indicates mean away from the target and also more variable

It is now clear that out of the three quality characteristics only in one instance namely total length, the process is capable of meeting the specifications but in the other two cases of thread length and the core diameter the process is incapable and defects are bound to occur. It may also be noted that even in the case of total length the minimum benchmark for Cp =1.33 is not satisfied. This means close control of the process is required to ensure that the target shift is well controlled and doesn't enlarge rapidly.

Now the question is what is the overall process capability considering all the three quality characteristics. It may be noted that all the three characteristics are formed on the same machine and the process is continuous. If only Cp values are used to indicate the overall quality, then all the three values have to be mentioned and an overall measure will not be possible. Secondly, even after looking at the three values an overall ranking of the process is not possible. While the process managers would appreciate a clear benchmarking of the overall process quality, the conventional process capability measures do not provide that important information.

Moving Towards Six Sigma Metrics

Six Sigma has now become a common philosophy or a technique to express the process quality. According to Brue (2002), six sigma can be applied to every type of organization for better process management. As stated by Devane (2004), six sigma's basic value proposition is that principle for process improvement, statistical methods, a customer focus, and a management system focusing on high-return improvement projects to result in continuous improvement and significant financial gains. Snee (2004) comments that six sigma can help in the growth of a company by properly deploying the process improvement methodology.The two metrics, namely, the Defects per million opportunities (DPMO) and the process sigma level, help the decision makers to quickly assess the overall process quality. Further, the process sigma level is currently used as a global benchmark to compare and understand the overall quality. As shown in the Table 5 it is possible to benchmark the processes using the DPMO as the measure of quality.

Table 5. Sigma levels and corresponding DPMO values

Sigma Performance Levels – One to Six Sigma	
Sigma Level	**Defects Per Million Opportunities (DPMO)**
1	690,000
2	308,537
3	66,807
4	6,210
5	233
6	3.4

If it is desired by the process managers to know how the conventionally used process metrics like percent defectives, DPMO values, percentage yield and the short term and the long term Cpk values then Table 6 can be used to conveniently interlink the various process capability measures to six sigma metrics. This has the advantage of providing both the old and new metrics being used to watch the process quality and also to assess the quality in terms of the six sigma metrics. It is to be observed here that there are two Cpk values shown as short term and long term values. The short-term Cpk value is greater than the long term Cpk by 0.5. This indicates the short term Cpk is better compared to long-term Cpk. This is because over a long term there will be a drift in the process mean which results in lowering the value of Cpk. This kind of a drift in the process mean is also accepted as normal behavior in Six Sigma calculations and it is customary to consider the drift as equal to 1.5 times the process standard deviation. Because of this drift the corresponding DPMO and the process sigma values also change.

Table 6. Sigma levels and corresponding conventional process measures

Sigma level	Sigma (with 1.5? shift)	DPMO	Percent defective	Percentage yield	Short-term Cpk	Long-term Cpk
1	-0.5	691,462	69%	31%	0.33	-0.17
2	0.5	308,538	31%	69%	0.67	0.17
3	1.5	66,807	6.7%	93.3%	1.00	0.5
4	2.5	6,210	0.62%	99.38%	1.33	0.83
5	3.5	233	0.023%	99.977%	1.67	1.17
6	4.5	3.4	0.00034%	99.99966%	2.00	1.5
7	5.5	0.019	0.0000019%	99.9999981%	2.33	1.83

Analysis and Discussion

The present case study is again taken up for analysis using the Six Sigma Metrics, particularly the DPMO and the process sigma level. Before that however one important parameter namely the number of defects needs to be ascertained. Looking at the data, it is now possible to find out how many defectives are there in the entire batch based on the instances of failure of a quality characteristic to meet the expected tolerance values. That is, any dimension that is either above the upper specification limit or below the lower specification limit is counted as an instance and accordingly for all the three characteristics the number of such occurrences are noted. These values are shown in Table 7 in which the number of instances greater than USL and less than LSL is extracted from the raw data.

In the next step the Six Sigma Metrics in terms of DPMO and the process sigma level are established. It is to be observed that a component can be a defective by virtue of one or more (in this case three) characteristics not satisfying the specifications. This is where the SSM in the form of DPMO offers an advantage over the conventional percent defective. As per the SSM methodology each instance is considered as an opportunity and the number of such instances not able to meet the specifications is counted. This means the same item may give rise to all the three instances which means there are three opportunities of non-compliance, but, only one defective. All the Six Sigma calculations have been done using the Excel software and displayed in the Table 8.

Table 7. Count of Defects based on deviations from the specifications

Specification	Total Length	Thread Length	Core Diameter
LSL	24.4	15	3.35
USL	25.4	16	3.45
	Values < LSL	Values >USL	Total
Total Length	1	1	2
Thread Length	10	0	10
Core Diameter	0	2	2

Table 8 DPMO values and Six Sigma level of the process for the three characteristics

	Total length	Thread length	Core diameter
Total defects	2	10	2
No. of opportunities	375	375	375
DPMO	5333.333333	26666.66667	5333.333333
Sigma Level	4.05343343	3.432212092	4.05343343

As seen in the Table 8, the SSM values are indicative of the process quality. However pertaining to three quality characteristics again it is possible to establish three independent SSM values corresponding to each of the quality characteristic. This indicates the quality of the process, but, not in one single measure rather as three sets of values. Thus the question arises as how to convert the same to a single measure applicable to the overall process quality.

Now to obtain a single measure, a different approach is followed. In this step to obtain a single measure of defective the defects due to different quality characteristics are considered and aggregated. It is to be noted that there are totally 14 instances of defects and consequently this is converted to the DPMO value and later to the Six Sigma level, thereby getting an overall measure of the process. This is displayed in the Table 9. Now, there is only one measure in terms of DPMO and process sigma level and thus conveniently the process capability is established and understood. The advantages of these two SSM values are quite obvious. How far away is the current process compared to the global benchmarks is quickly realized by referring to Tables 5 and 6 and motivates developing appropriate corrective actions to be deployed. Now it is obvious the process values have been combined into one single value which enables comparison and assessment very easily. World class companies will have a process sigma level anywhere above 5 and a DPMO of no more than 200. With this benchmark it is now easier to comment on the overall quality of the process on hand.

Further, how sensitive these SSM values are when subject to changes in process parameters needs to be ascertained. It is well established in the statistical quality control literature that any change in the process parameters is well detected by control charts for variables (mean, range, and standard deviation) as per the parameter being monitored. Once the assessment shifts to the attribute based quality control using the counts and not the measurements, the sensitivity of the measure to detect quick and minor changes diminishes. For example it is commonly stated that control charts for attributes namely p-charts, and c-charts, are not meant for precision in detecting the changes in process parameters. However, the biggest advantage is instead of several measurements being made and charts maintained separately, one single basis namely whether a given item is defective or not sums up the quality of the process disregarding the number of quality characteristics.

Table 9. Six Sigma metrics for the overall process

	Overall values
Total defects	14
No. of opportunities	375
DPMO	37333.33333
Sigma Level	3.282506534

Conclusion

Process capability measures play an important role in ascertaining the overall quality of the process and decision makers appreciate a single measure to use it for comparison and benchmarking against the global standards. This paper has demonstrated how it is possible to obtain the overall quality measure using SSM methodology instead of using the traditional process capability measures which while serves the purpose, if only, one characteristic is considered at a time. But, for a multiple characteristic situation the SSM approach seems to be better and helpful to decision makers. Further as the simulated results show they serve the purpose under different situations. However, it needs to be observed here that the approach suggested in this paper moves from variable measurement to attribute measurement. This to some extent diminishes the sensitivity of the measure in the case of smaller quick changes that my happen over a period of time. This incidentally happens in all those cases where the assessment shifts from variable measurement to attribute measure. Even control charts are no exception when X-bar and R charts are compared with respect to the P chart and the C charts. But the advantage of arriving at a single measure without much complications or resorting to exotic mathematical modeling cannot be ignored. Hence it is suggested that as long as precision is not given a higher weightage conveniently the SSM based process capability assessment can be used and can be extended for business negotiation purposes as well.

This paper offers scope for further work. How sensitive and accurate are the SSM values in reflecting the changes in process parameters when compared against the changes reflected by the values of process capability indices needs to be ascertained. The author intends to carry out this work using simulation modeling. However, the inherent limitations of normality assumption for the process output continue to exist in all these extensions of the six sigma approach.

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