Application of Non-Gaussian Distribution Models to Measure Value at Risk in Nifty Sectoral Indices

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Abstract

The aim of the study is to empirically investigate the performance of the normal, logistic, Hypersecant,

Laplace, and Cauchy distribution models in terms of value at risk management in NSE sectoral indices. Ten years of daily closing rates over the period of January 2010 to December 2020, for a total of 2730 observations, have been analysed. Models with steeper peaks and thicker tails than the normal distribution have been shown to capture key characteristics of financial markets more accurately. The best VaR estimates come from the Laplace and Hypersecant distributions.

Keywords: *Gaussian models; non-Gaussian dynamic models; value-at-risk models; financialrisk management; market risk.*

Introduction

A critical method for financial risk management is normalizing the value at risk (VaR). They need to measure the estimated amount of loss to the portfolio for a specific holding period at agiven confidence level (Sarykalin, 2008). Since the Basel Committee on Bank Supervision allows banks to meet capital requirements based on VaR estimates, enabling banks to use internal models for VaR calculation, the assessment has become a simple market risk management method (Chockalingam, 2018).

Financial engineering incorporates elements of risk assessment and risk management. At the individual and institutional level, the main goal is to optimize one's anticipated return while remaining within reasonable risk limits. Eliminating risk is unlikely, but we settle for boundingrisk at a 99% level, ensuring that the assets on hand are adequate to meet all contingencies witha likelihood of 0.99. These probability computations based on historical statistics and, as such,cannot be considered systematic (Han, 2004). Crises are triggered not by run-of-the-mill incidents but by unusual events—sometimes by a confluence of extreme events referred to as a perfect storm. Hence, an essential aspect of financial engineering is estimating tail probabilities, probabilities of relatively uncommon events. Current methods for fitting probability distributions to observed data are not well equipped for estimating probabilities ofextreme values. The challenge will provide opportunities for control engineers and scientists (Acharya, 2009), (Neftci, 2000).

In Finance, the idea of "Value at Risk" is used to attribute risk to a position or portfolio. The 1percent VaR is the 99th percentile of a probability distribution or distribution function and the1st percentile of a complementary distribution function (Ball, 2006). Suppose a portfolio of shares has a 1 percent regular

VaR of $1 million. In that case, the likelihood is 0.01 that the portfolio will lose more than $1 million in value over a single day if no trades occur. Quantifying VaR is essential for many financial institutions (Penza, 2011).

The Market Risk function usually calculates from historical observations. This method faces many drawbacks, such as insufficient sample numbers, improper modelling assumptions, and inaccurate performance (Trenca, 2011), (Sharma, 2012).

When using historical information, one usually fits a Gaussian distribution to the data.However, it is essential to have a good fit for the "tail" of the distribution because it is most critical to measure risk correctly (McNeil, 2000).

Much of investment management focus on full markets and the Black-Scholes formula. A complete market is one in which hedging strategies cannot effectively reproduce the market's price fluctuations. The assumption that asset prices follow a log-normal distribution, orGaussian distribution, concludes that regular fluctuations are normal. After all, real asset pricesdo not behave in the same manner as log-normal distributions (Jondeau, 2007), (Teneng, 2011).

Asset returns can see to adopt a "stable" distribution over long period. Each stable distributionhas a related α that ranges from 0 to 2. The Gaussian is the only distribution with a finite variance that is not necessarily complex or chaotic. All other bimodal distributions have a lowerbound of < 2 and infinite variance. If α < 1, then the mean may be infinite in a financial data set. However, this condition seldom occurs. Real movements in asset values are said to be heavy-tailed. Moreover, as seen in the graph to the right, real asset movements are best represented by stable distributions of α far below the critical value of 2. When α is small, the tails decay more slowly, and as α increases, the error in Gaussian approximations increases (Kimball, 2000), (Makridakis, 2010).

Averages of heavy-tailed random variables also obey the law of large numbers (the average converges in probability to the real mean as the number of samples increases). However, do not follow the central limit theorem (fluctuations about the real mean are not necessarily Gaussian). Broad abnormal returns are more "burst" in terms of heavy-tailed distributions. In short, unusual events do not occur as infrequently as log-normal models say. Broad swings (tenor more standard deviations when log-normal approximations use) are more common than is indicated by the typical log-normal model (Vidyasagar, 2010), (Bianchi, 2019).

We initially evaluate the Value-at-Risk forecastability of a broader selection of sectoral indices in the context of non-gaussian distributions, such as Normal, Logistic, Hypersecant, Laplace, and Cauchy, for the daily log returns. Additionally, we present a method for generating a forecastability indicator (with a predetermined probability) by employing a series of confidence interval tests.

Methodology

An empirical distribution function of stock returns how to apply those distributions to model these stock returns and determine whether they fit decently. Whether the fit is terrible so in other terms, which distribution does fit the data the best, But the practical purpose of all these.sometimes challenging mathematical tasks well one of the most straightforward applications of distributions to practical finance calculations or considerations is Value Risk.

Investors assume that the portfolio returns followed a particular distribution and investigated itmostly using the normality assumption. However, most of the research discussed extensively in the introduction chapter points out that the normality assumption often violates.

If the researcher wants to apply parametric value at risk, for example, variance-covariance, orknown by many names, a researcher might need to consider another distribution rather than normal. Apply in five distinct distributions, four of them non-normal, featuring various degreesof heavy or fat tails to calculating value at risk. For various thresholds to do that mathematically, first need to derive the circled quantile function for each distribution function.Start with the cumulative distribution function in each of those four cases, and those distribution functions might seem familiar. What we need today is to calculate or derive the inverse function as the cumulative distribution function gives us the probability that an absolute value of x will not exceed by a random variable drawn from that distribution. For value at risk purposes, we need to know the inverse we need to know given a particular threshold of the loss that we will achieve in the end percent of worst-case scenarios.

Here functional relationship where x is our independent variable and $f(x)$ is our dependent variable. However, we need the inverse we need to derive the function where we express x. Hence, our return in that case in terms of $f(x)$, so in our case, probability or VaR the worst- case percentage scenarios. Hence, in each of those four distribution functions, we need to applysome mathematical transformations to derive the inverse function.

Normal Distribution

The Normal distribution is a good approximation to many other distributions for various applications due in part to the Central Limit Theorem, making it a good approximation to manyother distributions.

Variations of a naturally occurring variable also display distributions that resemble a "normal distribution." Examples include adult height, arm span. The population data generally conformto a straight line, but there is a bit more density near the tails.

The normal inverse function is defined in terms of the normal cdf as

x=F^(-1) (p|μ,σ)={x:F(x|μ,σ)=p},

Logistic distribution

Logistic distributions are often used in demographic and economic modelling because they aresimilar to the standard distribution but appear to have a slightly higher peak. The term does notoften appear in risk analysis models.

The cumulative function use as a basis for a 'growth curve' whose distribution function allowsfor being modelled as a Sech-Squared distribution. It derives the limiting distribution as n reaches infinity of the uniform mid-range (average of the maximum and lowest values) of a random sample of size n from an exponential-type distribution.

Inverse Cumulative Distribution Function

$$F(x) = \frac{1}{1 + e^{-\frac{x-\mu}{\alpha}}}$$

$$\frac{1}{F(x)} - 1 = e^{-\frac{x-\mu}{\alpha}}$$

$$\frac{1 - F(x)}{F(x)} = e^{-\frac{x-\mu}{\alpha}}$$

$$\frac{F(x)}{1 - F(x)} = e^{\frac{x-\mu}{\alpha}}$$

$$\frac{F(x)}{1 - F(x)} = e^{\frac{x-\mu}{\alpha}}$$

$$\ln\left(\frac{F(x)}{1-F(x)}\right)=\frac{x-\mu}{a}$$

$$x=\mu+a\ln\left(\frac{F(x)}{1-F(x)}\right)$$

Hyperbolic-Secant distribution

The Hyperbolic-Secant distribution is asymmetric distribution similar to the Regular distribution with increased peakedness, so it is called a more extreme variant of the Normal. Examples of the Hyperbolic-Secant distribution are shown below with their mean and standarddeviation and the standard Normal distribution.

The Hyperbolic-Secant distribution use to suit data that tend to be roughly Normal in distribution but show smaller shoulders, just as the Student Distribution and the Generalized Error Distribution are alternatives for data that appear to be larger than a Normal.

Inverse Cumulative Distribution Function

$$F(x)=\frac{2}{\pi}\arctan\left(e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right)$$

$$\frac{\pi}{2}F(x)=\arctan\left(e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right)$$

$$\tan\left(\frac{\pi}{2}F(x)\right)=e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}$$

$$\ln\left(\tan\left(\frac{\pi}{2}F(x)\right)\right)=\frac{\pi}{2}\frac{x-\mu}{\sigma}$$

$$x=\mu+\frac{2\sigma}{\pi}\ln\left(\tan\left(\frac{\pi}{2}F(x)\right)\right)$$

Laplace distribution

X and Y are two equivalent exponential (s) distributions which differ by m units when shiftedm to the right (m, s). The Laplace distribution in the figure shares the same form as a normal distribution, but has a sharper apex and longer tails than a Normal distribution.

The Laplace distribution use in several particular applications, but almost all of them refer backto the fact that it has relatively long tails instead of the Normal distribution. It has recently become widespread in modelling financial variables (Brownian-Laplace motion) due to the greater likelihood of extreme values. They provide a detailed study of the Laplace distributionin the monograph of (Kotz, 2001).

Inverse Cumulative Distribution Function

$$F(x) = \frac{1}{2}e^{\frac{x-\mu}{b}}, x \leq \mu$$

$$2F(x) = e^{\frac{x-\mu}{b}}$$

$$\ln(2F(x)) = \frac{x-\mu}{b}$$

$$x = \mu + \mathrm{bln}(2F(x))$$

Cauchy distribution

The Cauchy distribution derives two independents Normal (0,1) distributions, respectively X and Y, where X/Y = Cauchy (0,1). The Cauchy (a, b) move to have a median at a and b timesthe spread of a Cauchy (0,1).

The Cauchy distribution rare to use in measuring risk. A semiconductor is used in many areas,including physical anthropology, electrical and mechanical theory, and calibration problems. In physics, Lorentzian distribution is what researchers define as the distribution of the energy of an unstable state in quantum mechanics. CFD simulation is used to predict the points of impact from the straight line of particles released from a point source. This Cauchy distributionis used mainly to demonstrate how 'smarter' you are with individual judgments.

Inverse Cumulative Distribution Function

$$F(x) = \frac{1}{\pi}\arctan\left(\frac{x-\mu}{\gamma}\right) + \frac{1}{2}$$

$$\pi F(x) - \frac{\pi}{2} = \arctan\left(\frac{x-\mu}{\gamma}\right)$$

$$\tan\left(\pi F(x) - \frac{\pi}{2}\right) = \frac{x-\mu}{\gamma}$$

$$x = \mu + \gamma \tan\left(\pi F(x) - \frac{\pi}{2}\right)$$

Data

This study uses the daily return of NSE Nifty fifty and selected sectoral indices close prices from NSE website (www.nseindia.com) over the period from January 2010 to December 2020.These close prices are converted to daily return Log Return series.

Table 3.1 ADF Unit Root Test Results (Daily Returns of NSE Sectoral Indices)

Indices				***t-Statistic***	***Probability***
Nifty 50	***Augmented Dickey-Fuller test statistic***			***-52.0156***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Auto					
	Augmented Dickey-Fuller test statistic			***-49.7029***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Bank	***Augmented Dickey-Fuller test statistic***			***-48.9721***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
FinancialService	***Augmented Dickey-Fuller test statistic***			***-49.6846***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
FMCG	***Augmented Dickey-Fuller test statistic***			***-52.6229***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Health Care	***Augmented Dickey-Fuller test statistic***			***-48.9407***	***0.0001***

	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
IT	***Augmented Dickey-Fuller test statistic***			***-52.8998***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Media	***Augmented Dickey-Fuller test statistic***			***-50.4713***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Pharma	***Augmented Dickey-Fuller test statistic***			***-49.9256***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Private bank	***Augmented Dickey-Fuller test statistic***			***-48.544***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	
Realty	***Augmented Dickey-Fuller test statistic***			***-47.6316***	***0.0001***
	Test critical values:	***1% level***		***-3.43256***	
		5% level		***-2.8624***	
		10% level		***-2.56727***	

A unit root test detects if a non-stationary time series variable has a unit root. The null hypothesis is the presence of a unit root, while the alternative hypothesis is stationarity, trend stationarity, or explosive root. Table 3.1 indicates that the unit root test shows that the first difference between the Nifty Fifty is all stationary.

Analysis and Discussion

For each index, the unconditional mean of daily returns is very close to zero. The unconditionalstandard deviation is especially high for Realty (0.021113). For the rest of stock index returnsthe standard deviation moves between 0.011019 (FMCG). The skewness statistic is negative and significant for all the indexes. This means that the distribution of those returns is skewed to the left. For all the indexes considered, the excess kurtosis statistic is very large and significant at 1% level implying that the distributions of those returns have much thicker tails than the normal distribution. Similarly, the Jarque-Bera statistic is significant rejecting the assumption of normality.

All of them find evidence that the empirical distribution of the financial return is asymmetric and exhibits a significantly excess of kurtosis (fat tails and peakness). In order to capture the non-normal characteristics observed in our data set, we fit several Parametric distributions: Logistic, Hypersecant, Laplace, Cauchy and Normal distributions. In Table 3.2 we present theestimated parameters of these distributions.

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Table 4.1. Descriptive Statistics for Daily Returns of Sectoral Indices of NSE

Sectoral Indices	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
Nifty 50	0.00036	0.000637	0.084003	-0.139038	0.011026	-1.01326	17.88492	25660.28	0
Auto	0.000389	0.00073	0.098997	-0.149055	0.013729	-0.43986	12.62099	10613.23	0
Bank	0.000452	0.000742	0.099951	-0.18313	0.015568	-0.64711	13.772	13384.73	0
Financial Service	0.000521	0.000799	0.089107	-0.173623	0.01475	-0.76955	13.91464	13815.35	0
FMCG	0.000566	0.000869	0.079906	-0.111998	0.011019	-0.36594	11.78087	8828.247	0
HealthCare	0.00047	0.000575	0.087976	-0.086927	0.01105	-0.3488	8.75701	3823.989	0
IT	0.000519	0.000627	0.08922	-0.124903	0.013319	-0.6733	12.39616	10245.25	0
Media	8.86E-06	0.000388	0.080423	-0.178817	0.015543	-0.91346	12.4026	10432.34	0
Pharma	0.000453	0.000586	0.09865	-0.093507	0.012033	-0.21809	8.445107	3392.994	0
Private Bank	0.000581	0.000663	0.104854	-0.196954	0.015723	-0.74188	16.32615	20443.36	0
Realty	-0.000179	0.0011	0.08093	-0.123348	0.021113	-0.46052	5.332165	714.9197	0

Note: This table presents the descriptive statistics of the daily percentage returns of NSE Sectoral Indices. The JB statistic is distributed as the Chi-square withtwo degrees of freedom. The value for kurtosis is greater than the Normal distribution value (±3), suggesting that the regular return distribution has characteristics of "sharp peak" and "fat tail." Study denies the null hypothesis that the return sequence Normally distributed.

Table 4.2 Maximum likelihood estimates of alternative distribution functions

Sectoral Indices	Normal		Logistic		Hypersecant		Laplace			Cauchy	
	μ	Σ	μ	Σ	μ	Σ	μ	σ	λ	μ	σ
Nifty 50	3.60E-04	0.01103	3.60E-04	0.00608	3.60E-04	0.01103	3.60E-04	0.01103	128.26	7.74E-04	0.00522
Auto	3.89E-04	0.01373	3.89E-04	0.00757	3.89E-04	0.01373	3.89E-04	0.01373	103.01	7.86E-04	0.00675
Bank	4.52E-04	0.01557	4.52E-04	0.00858	4.52E-04	0.01557	4.52E-04	0.01557	90.841	7.85E-04	0.00721
Financial Service	5.21E-04	0.01475	5.21E-04	0.00813	5.21E-04	0.01475	5.21E-04	0.01475	95.879	7.95E-04	0.00685
FMCG	5.66E-04	0.01102	5.66E-04	0.00608	5.66E-04	0.01102	5.66E-04	0.01102	128.34	8.44E-04	0.00532
HealthCare	4.70E-04	0.01105	4.70E-04	0.00609	4.70E-04	0.01105	4.70E-04	0.01105	127.98	7.45E-04	0.00552
IT	5.19E-04	0.01332	5.19E-04	0.00734	5.19E-04	0.01332	5.19E-04	0.01332	106.18	7.08E-04	0.00618
Media	8.86E-06	0.01554	8.86E-06	0.00857	8.86E-06	0.01554	8.86E-06	0.01554	90.988	6.51E-04	0.00777
Pharma	4.53E-04	0.01203	4.53E-04	0.00663	4.53E-04	0.01203	4.53E-04	0.01203	117.53	6.49E-04	0.00598
Private Bank	5.81E-04	0.01572	5.81E-04	0.00867	5.81E-04	0.01572	5.81E-04	0.01572	89.948	7.18E-04	0.00722
Realty	-1.79E-04	0.02111	-1.79E-04	0.01164	-1.79E-04	0.02111	-1.79E-04	0.02111	66.984	0.0013	0.01093

Note: This table provides the estimates for the mean (μ) and the standard deviation (σ) of log-return. As expected, these estimates are quite similar across distributions and do notdiffer much from the simple arithmetic means and standard deviations of log-returns.

Comparison of the distributions in statistical terms

In this section we want to answer the following question: Which distribution is the best one forfitting asset returns? The above results provide strong support to the hypothesis that stock returns are not normal.

Kolmogorov-Smirnov Goodness-of-Fit Test

The Kolmogorov-Smirnov test is used to decide if a sample comes from a population with a specific distribution.

The Kolmogorov-Smirnov test is defined by:

Null: The data follow a specified distribution Alternative: The data do not follow the specified distribution

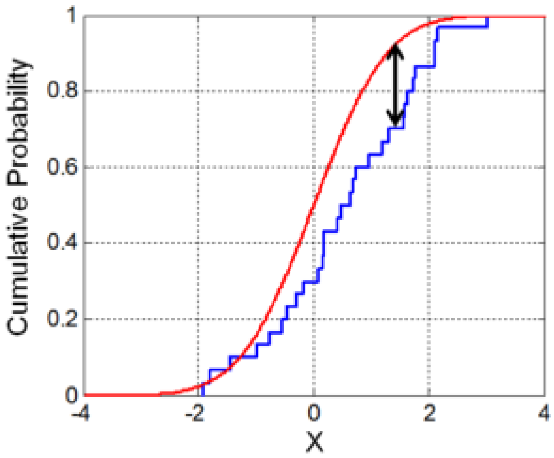

Figure 4.1 Kolmogorov–Smirnov statistic

Illustration of the Kolmogorov–Smirnov statistic. The red line is a model CDF, the blue line isan empirical CDF, and the black arrow is the K–S statistic.

If researchers are using a technique that makes a normality (or some other type of distributional) assumption, it is important to confirm that this assumption is in fact justified. Ifit is, the more powerful parametric techniques can be used. If the distributional assumption is not justified, using a non-parametric or robust technique may be required.

In order to enhance the robustness of the fitting results, we performed the Kolmogorov– Smirnov test for the goodness-of-fit test. According to the test results given in Tables. we candetermine which theoretical distribution differs significantly from the given return distributionfor each stock index. Based on the Kolmogorov–Smirnov test, we determined that the distributions in bold are able to describe the return distribution with the given significance level.

The Kolmogorov–Smirnov test uses the whole samples to calculate the statistics, which represent the maximum difference value between the empirical distribution function and the theoretical distribution function. However, in extreme cases, the left and the right tails of the return distribution are usually affected in terms of risk management. In other words, the tails of the return distribution and the risk management are interrelated.

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Table 4.3 Goodness-of-fit tests

Sectoral indices	Sample Size	Normal		Logistic		Hypersecant		Laplace		Cauchy	
		Statistic	P-Value	Statistic	P-Value	Statistic	P-Value	Statistic	P-Value	Statistic	P-Value
Nifty 50	2729	0.06412	3.33E-10	0.04592	1.93E-05	0.03185	0.00772	0.0304	0.01265*	0.05807	1.90E-08
Auto	2729	0.05537	1.02E-07	0.0328	5.51E-03	0.02093	0.1803***	0.03112	0.0099*	0.05765	2.49E-08
Bank	272	0.06444	2.65E-10	0.04372	5.68E-05	0.02956	0.01663*	0.02209	0.13741***	0.05024	1.99E-06
Financial Service	2729	0.06595	9.03E-11	0.04563	2.23E-05	0.03134	0.00919	0.02508	0.06353***	0.05069	1.54E-06
FMCG	2729	0.05832	1.62E-08	0.03698	0.00112	0.02185	0.14535***	0.03065	0.0116*	0.05627	5.87E-08
HealthCare	2729	0.05153	9.67E-07	0.03336	0.0045	0.022	0.1404***	0.03457	0.00287	0.0595	7.59E-09
IT	2729	0.06626	7.21E-11	0.04574	2.11E-05	0.03146	0.00882	0.02255	0.12282***	0.0555	9.43E-08
Media	2729	0.05836	1.58E-08	0.04039	2.64E-04	0.02765	0.03027**	0.03995	3.20E-04	0.06059	3.70E-09
Pharma	2729	0.05255	5.42E-07	0.03233	0.00652	0.02039	0.20387***	0.0274	0.03264**	0.0577	2.41E-08
Private Bank	2729	0.06795	2.09E-11	0.04625	1.64E-05	0.03078	0.01112*	0.01626	0.46131***	0.04983	2.48E-06
Realty	2729	0.04985	2.45E-06	0.03534	0.00214	0.03148	0.00875	0.05541	9.97E-08	0.06376	4.28E-10

Note: Significance levels, 0.05***,0.02** and 0.01* shows not rejecting the null hypothesis.

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Looking to each figure, we chose the best-fit candidate distributions for the left tail in the returndistribution for each stock index. Therefore, Laplace distribution fits in Nifty 50(1%), Auto (1%), Bank (all levels), Financial Service (all levels) FMCG (1%), IT (all levels), Pharma (1%and 2%), and Private Bank (all levels). Hypersecant distribution fits in Auto (all levels), Bank(1%), FMCG (all levels), Healthcare (all levels), Media (1% and 2%), Pharma (all levels), and Private Bank (1%). Logistic distribution not fit in any sectoral indices but it comes second bestdistribution after the right distribution fit. None of the distributions fits in Nifty Realty Sector.Normal and Cauchy distributions not fit in any sectoral indices but its highly useful in terms ofVaR Comparison among other distributions.

Evaluating the performance in terms of VaR

In this section we compare the normal and the skewed distributions in terms of VaR. The comparison is carried out evaluating (i) the accuracy of the VaR estimates and (ii) the losses that VaR produces. For each distribution, we use parametric approaches to forecast the VaR out-of-the-sample one-step-ahead at 1%, 0.25% and 0.5% confidence level.

Table 4.4 Ratio of VaR α=0.1% for each VaR model across NSE Sectoral Indices

	Normal	*Logistic*	*Hypersecant*	*Laplace*	*Cauchy*
Nifty 50	*-2.53%*	*-2.76%*	*-2.88%*	*-4.28%*	*-16.53%*
Auto	*-3.16%*	*-3.44%*	*-3.59%*	*-5.33%*	*-21.40%*
Bank	*-3.58%*	*-3.90%*	*-4.07%*	*-6.05%*	*-22.86%*
Financial Service	*-3.38%*	*-3.68%*	*-3.85%*	*-5.72%*	*-21.72%*
FMCG	*-2.51%*	*-2.74%*	*-2.86%*	*-4.25%*	*-16.84%*
HealthCare	*-2.52%*	*-2.75%*	*-2.87%*	*-4.28%*	*-17.49%*
IT	*-3.05%*	*-3.32%*	*-3.47%*	*-5.16%*	*-19.59%*
Media	*-3.61%*	*-3.94%*	*-4.11%*	*-6.08%*	*-24.66%*
Pharma	*-2.75%*	*-3.00%*	*-3.14%*	*-4.66%*	*-18.96%*
Private Bank	*-3.60%*	*-3.93%*	*-4.10%*	*-6.09%*	*-22.90%*
Realty	*-4.93%*	*-5.37%*	*-5.60%*	*-8.28%*	*-34.65%*

Note: Bold figures indicate the Most favoured models.

Table 4.4 indicates that Laplace distribution is considered as the most favoured model in highlyvolatile indices like Nifty 50, Auto, Bank, Financial Services, FMCG, IT, Pharma, Private Bank and PSU Bank. The Hypersecant distribution also best model in Estimation of VaR in Sectors like Auto, Bank, Consumer Durables, FMCG, Healthcare, Media, Pharma, and PrivateBank. The results of Normal and Cauchy distribution reveal the Least favoured models in VaREstimation.

Table 4.5 Ratio of VaR α=0.25% for each VaR model across NSE Sectoral Indices

	Normal	***Logistic***	***Hypersecant***	***Laplace***	***Cauchy***
Nifty 50	***-3.06%***	***-3.61%***	***-3.85%***	***-5.81%***	***-66.38%***
Auto	***-3.82%***	***-4.49%***	***-4.80%***	***-7.24%***	***-85.86%***
Bank	***-4.33%***	***-5.09%***	***-5.45%***	***-8.20%***	***-91.72%***
Financial Service	***-4.09%***	***-4.82%***	***-5.15%***	***-7.76%***	***-87.14%***
FMCG	***-3.04%***	***-3.58%***	***-3.83%***	***-5.78%***	***-67.65%***
HealthCare	***-3.05%***	***-3.60%***	***-3.85%***	***-5.81%***	***-70.21%***
IT	***-3.69%***	***-4.34%***	***-4.65%***	***-7.01%***	***-78.61%***
Media	***-4.36%***	***-5.13%***	***-5.48%***	***-8.23%***	***-98.86%***
Pharma	***-3.33%***	***-3.93%***	***-4.20%***	***-6.33%***	***-76.07%***
Private Bank	***-4.35%***	***-5.13%***	***-5.49%***	***-8.27%***	***-91.85%***
Realty	***-5.94%***	***-6.99%***	***-7.46%***	***-11.20%***	***-139.03%***

Note: Bold figures indicate the Most favoured models.

Table 4.5 indicates that Laplace distribution is considered as the most favoured model in extremely unpredictable indices like Bank, Financial Services, IT, Pharma and Private Bank. The Hypersecant distribution also best model in Estimation of VaR in Sectors like Auto, FMCG, Healthcare, Media, and Pharma.

Table 4.6 Ratio of VaR α=0.5% for each VaR model across NSE Sectoral Indices

	Normal	***Logistic***	***Hypersecant***	***Laplace***	***Cauchy***
Nifty 50	***-2.81%***	***-3.18%***	***-3.37%***	***-5.04%***	***-33.15%***
Auto	***-3.50%***	***-3.97%***	***-4.20%***	***-6.28%***	***-42.89%***
Bank	***-3.97%***	***-4.50%***	***-4.76%***	***-7.13%***	***-45.82%***
FinancialService	***-3.75%***	***-4.25%***	***-4.50%***	***-6.74%***	***-43.53%***
FMCG	***-2.78%***	***-3.16%***	***-3.34%***	***-5.02%***	***-33.78%***
HealthCare	***-2.80%***	***-3.18%***	***-3.36%***	***-5.04%***	***-35.06%***
IT	***-3.38%***	***-3.83%***	***-4.06%***	***-6.08%***	***-39.27%***
Media	***-4.00%***	***-4.54%***	***-4.79%***	***-7.16%***	***-49.40%***
Pharma	***-3.05%***	***-3.46%***	***-3.67%***	***-5.49%***	***-38.00%***
Private Bank	***-3.99%***	***-4.53%***	***-4.79%***	***-7.18%***	***-45.89%***
Realty	***-5.46%***	***-6.18%***	***-6.53%***	***-9.74%***	***-69.45%***

Note: Bold figures indicate the Most favoured models.

Table 4.6 indicates that Laplace distribution is considered as the most favoured model indiceslike Bank, Financial Services, IT, Pharma and Private Bank. The Hypersecant distribution also best model in Estimation of VaR in Sectors like Auto, FMCG, Healthcare, and Pharma.

Overall study concludes that Normal distribution performs very poor in estimating VaR (this distributionunderestimate risk in almost all indices). After the normal distribution the Logistic distribution also poorly performedexcept in two indices Hypersecant and Laplace distributions considered as most favoured distributions and estimates VaR most precisely manner. Cauchy distribution advocate unlimited risk in VaR Estimation.

Conclusion

This section evaluates the performance of several skewed and symmetric distributions in modelling the tail behavior of daily returns and in forecasting VaR. The parametric distributions considered are Normal, Logistic, Hypersecant, Laplace and Cauchy distributions.

For this study we have used daily returns of NSE Sectoral Indices: Nifty 50, Auto, Bank, Consumer Durables, Financial Service, FMCG, HealthCare, IT, Media, Oil and Gas, Pharma,Private Bank, PSU bank and Realty. The sample used for the statistical analysis runs from 1st January 2010 to 31st December 2020.

From the results presented in the chapter, we concluded that the skewness and fat tail distributions outperform the normal one in fitting financial returns and forecasting VaR. Among all the Parametric distributions considered in this paper, Laplace distribution and Hypersecant distribution estimate the best VaR results (Taylor, 2019). The normal distributionis the least performed and underestimated risk. Cauchy distribution estimates the unlimited orworst loss occur in the VaR analysis (Alzaatreh, 2016).

Extreme occurrences do occur on occasion, even in generally conservative risk models. By definition, extreme events are outliers. Because it is hard to eliminate risk when investing, therisk from extreme events is moved elsewhere and compensated for with reduced projected profits. Studying these events and statistically modelling them is beneficial when the knowledge successfully integrates into the overall system without affecting the other elementsof the study. Separate assessments of a distribution's outliers and centre values are likely to getboth parts inaccurate if not done correctly.

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