

Option price modelling with Ito Lemma Model and Heston approach

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ABSTRACT

We provide an introduction to risk management with the use of derivatives, especially options. We further explore options that are traded in NSE, and we analyse the behaviour of call options with the binomial model. For Bank Nifty options, the Black-Scholes model with available data of daily price of underlying asset, strike price, assumed interest rate and time to expiry, pricing is analysed and simple simulation is undertaken under different conditions using R. An attempt is made to analyse call option pricing with Ito lemma model for Bank Nifty options for Jun 2026. With an introduction to the Heston stochastic volatility model, simulation modelling is undertaken. We focus on developing mathematical formula as a tool for stimulating further interest and research in financial mathematics.

Key words: *Option, Black Scholes, Heston, simulation*

Introduction

An option is a contract that allows the holder the right to buy or sell an underlying asset or financial instrument at a specified strike price on or before a specified date, depending on the type of the option. An option that gives the holder the right to buy at a specified price is called a call option, while one that gives the right to sell at a specified price is called a put option. An option can be American or European option depending on the exercise time. A European option gives the holder the right to exercise the option only on the expiration date. An American option gives the holder the right to exercise the option at any time from the date of purchase up to and including the expiration date. Selling or exercising an option before the expiry date typically requires a buyer to pick the contract up at the determined price. The strike price may be set by reference to the spot price (market price) of the underlying security or commodity on the day an option is issued, or it may be fixed at a discount/premium. The issuer has the corresponding obligation to fulfil the transaction (to sell or buy) if the holder "exercises" the option. The payoff of a call option at expiration is $= \max (S_t - K, 0)$. S_t denotes the market price of the underlying asset and K denotes the strike price.

The price of a call option depends on the current market price of the underlying asset, strike price, time to expiration, volatility of the underlying asset, risk-free interest rate. Black Scholes formula (1973) considers all these factors in assessing call option price. The current market price of the underlying asset has a positive relationship with the call option price, because an increase in the underlying asset price increases the potential gain from exercising the call option. The strike price has an inverse relationship with the call option price, because a higher strike price makes it less attractive to purchase the underlying asset through the option. The time to expiration generally has

a positive effect on the call option price because a longer period provides greater opportunity for the underlying asset price to move above the strike price. Volatility has a positive relationship with the call option price because higher volatility increases the probability of large favourable movements in the underlying asset price. The risk-free interest rate generally has a positive effect on the value of a call option because a higher interest rate reduces the present value of the future payment of the strike price. Expected dividends generally have a negative effect on the call option price because dividend payments reduce the expected future price of the underlying asset. Therefore, the major factors determining the price of a call option are underlying asset price, strike price, time to expiration, volatility, risk-free interest rate, and expected dividends. Many models are proposed to assess the value of call option hence the value of put option using the data of dependent variables.

Literature Review

Falloul and Falloul (2015) examined the development of an option-pricing framework based on the Black–Scholes model. Their study connects geometric Brownian motion, Itô's lemma, the Black–Scholes partial differential equation, and numerical option pricing. The underlying asset is represented through a stochastic differential equation.

El Euch and Rosenbaum (2019) work on the rough Heston model represents an important development in stochastic volatility modelling. Rough-volatility models were motivated by empirical evidence that financial volatility exhibits much more irregular and short-memory behaviour than traditional stochastic-volatility models. The rough Heston framework uses a fractional/rough stochastic structure and provides a characteristic function that can be used for European option pricing. The resulting framework allows researchers to calculate call-option prices using Fourier-based methods while capturing features of observed volatility that are difficult to reproduce using constant-volatility Black–Scholes model

Popoola Odunayo Emmanuel and Oluwabunmi Veronica Kehinde-Dada i(2024) studied behaviour of the correction of Black-Scholes portfolios based on historical stock price data by building we build a model that simulates one sample path of the stock price stochastic process at discrete time steps and track the correction over a time interval as it relates to the change in stock price over time.

Washburn and Dik (2020) present a simplified derivation of the Black-Scholes equation using Itô's Lemma, with emphasis on the intuition behind the mathematical processes involved. The study explains how a hedged portfolio can be constructed to eliminate risk and derive the Black-Scholes partial differential equation for option pricing. The authors note that difficulties in understanding the derivation often arise from mathematical notation and a lack of intuition about the underlying mathematical processes. Their work provides a useful foundation for understanding classical option pricing and serves as a basis for extending option-pricing analysis to advanced stochastic volatility models such as the Heston model.

Other Methods Besides Finite Difference Methods: There are many methods for solving Partial Differential Equations (Zauderer, 2006). Many of those approaches, specifically, Laplace and Fourier transforms, expansions in Fourier and Taylor series, have been used for option pricing (Heston, 1993; Carr and Madan, 1999; Duffie et al., 2000; Davydov and Linetsky, 2003). After studying these articles, it can be seen that option price models are based on mathematical and stochastic differential equations and there is a further scope of testing models on real data available from market. There exists gap for further research on option price modelling.

Research design:

The call option on banknifty index is considered in the first stage. The values of call option for two strike price 58000 and 60000 is considered. The data collected over June 1 to June 29 and the strike date is June 29, 2026.

Data Analysis:

Data of call and put option on NSE banknifty is considered from 1 June to 29 June 2026 from NSE website, covering a total 19 working days. The data of LTP (Last traded price) at the end of working hours of each working day is considered.

Table no 1 when strike price is 58000

	K= 58000			
Day	Date	Call	Put	Bank nifty
1	Jun '1	88.7	3990	53643
2	Jun '2	95	4285	53714
3	Jun '3	169	4100	54185
4	Jun '4	140	3140	54307
5	Jun '5	140	3399	54496
6	Jun '8	172	2700	54063
7	Jun '9	190	2783	55249
8	Jun'10	125	2981	54958
9	Jun'11	137	3500	55100
10	Jun'12	137	3303	56813
11	Jun'15	418	1195	57200
12	Jun'16	384	1048	57297
13	Jun'18	545	516	57963
14	Jun'19	413	586	57685
15	Jun'22	418	429	57935
16	Jun'23	159	988	57183
17	Jun'24	480	285	58150
18	Jun'25	433	185	58177
19	Jun'29	153	300	57727

(Source: NSE India, NIFTY Options Historical Data) Table no.2 when strike price is 60000



	K =60000			
Day	Date	Call	Put	Bank nifty
1	Jun '1	28	5990	53643
2	Jun '2	37	5591	53714
3	Jun '3	38	5060	54185
4	Jun '4	37	5108	54307
5	Jun '5	34	4963	54496
6	Jun '8	37	4563	54063
7	Jun '9	31	4393	55249
8	Jun'10	288	4866	54958
9	Jun'11	28	4777	55100
10	Jun'12	25	4685	56813
11	Jun'15	63	2840	57200
12	Jun'16	47	2732	57297
13	Jun'18	52	2030	57963
14	Jun'19	28	2210	57685
15	Jun'22	20	2039	57935
16	Jun'23	12	2843	57183
17	Jun'24	17	1823	58150
18	Jun'25	9	1740	58177
19	Jun'29	4	2152	57727

(Source: NSE India, NIFTY Options historical data)

The options for strike price of 58000 with a strike date of 29th June are considered under the study period for June 1 to 29, 2026

Average Daily Return: +0.4142% per day, Daily Volatility (Sample Standard Deviation):

1.1043% and assume a risk-free rate 5% p.a.

The daily volatility of Nifty 50 returns for previous 6 month period (Jan1 - Jun 30 2026) differs significantly. Number of Trading Observations: 116, Mean Daily Return: -0.02%, Daily Volatility (Standard Deviation): 1.04% and annualised volatility: 12.4% as per data^a

Binomial model:

Using risk-free rate $r = 5\%$ and if the asset price moves up by a small amount it is denoted as u and similar decrease as d . S_t denotes future asset value. S is current asset value. And if

$u = 2\%$ and $d = -2\%$ for 10 days iteration, $R = 1.05^{1/252} = 1.00019363$ and $p = (R - d)/(u - d)$ So $p = 0.504$ and S_t formula is given as

$$S_t = S (u)^j (d)^{10-j} = 54958 (u)^j (d)^{10-j}$$

For $j = 0$ to 10 then S_t is 44904 to 66,993.

The binomial model indicates that the theoretical value of the call option decreases as the strike price increases. When the strike price is ₹58,000, the theoretical call value is approximately

₹427.37. When the strike price increases to ₹60,000, the theoretical call value decreases to approximately ₹139.06. Similarly, simulation can be done to determine call values for strike prices of 58000 and 60000 for different u and d values. For $u = d = 1\%$, the call price is Rs 36 for strike price 58000 and Rs 0.84 for strike price 60000.

Black Scholes model:

On day June 10th the Nifty 50 is 54958. With the given data and assumption of the risk-free rate and volatility assuming volatility, the call price can be determined using the Black-Scholes model (BS model) formula. C denotes Call option price and K denotes the strike price t is time to expiry and r is the risk-free interest rate.

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$\text{where, } d_1 = [\ln(S_0/K) + (r + \sigma^2/2)T] / (\sigma\sqrt{T}) \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

Therefore, the option price P can be determined using put-call parity. The put-call parity is given as $P = C - S_0 + Ke^{-rT}$

On 10th June, with $S = 54958$ for strike price of 58000, the BS model returns a call price of Rs 22 and a put price of Rs 2913, which differ significantly from the actual values of Rs 125 and Rs 4866 for the call and put options, respectively. Similarly, for strike price Rs 60000, the values are Rs 0.6 and Rs 4866 which differ significantly from actual values of Rs 288 and Rs 4866 of call and put respectively. Volatility differs daily.

Similarly, on June 29th asset price for $K = 58000$ is found to be 57727 and call price should be $\text{Max}(S - E, 0) = \text{Max}(57727 - 58000, 0) = 0$. But the actual data is Rs 153 and for $K = 60000$ call value should be zero whereas the actual value is Rs 4.

Weiner process and Ito Lemma model

The Wiener model can be used for analysis. A general Wiener process for variable x can be defined in terms of dz as $dx = a dt + b dz$. Here, the $a dt$ term implies that x has an expected drift rate of a per unit of time. $b dz$ reflects a Wiener process. The same can be used to reflect asset price S as follows:

$$dS = \mu S dt + \sigma S dz$$

$$\Delta S = \mu S \Delta t + \sigma \epsilon (\Delta t)^{1/2}$$

As on June 10th, Bank Nifty is 56813, and set $S_0 = 56813$ and $\mu = 5\%$ and $\Delta t = 1/365$ and $\sigma = 177\%$
So on June 29th day using R code simulation

$$S_1 = 56813 + 7.7826 + 32.0271\epsilon$$

Using R code simulation 95% simulation prediction range S_t ranges from 57692 to 58107

Using BS model Call value can be derived. Payoff = $\max(S - E, 0)$. When S_t is 56692 call payoff

= $\max(56692 - 57000, 0)$ which results rupees zero and S_t is 58107 call payoff = $\max(58107 - 58000, 0)$ which results in Rs 107

Heston Call Option Pricing Formula

The Heston model assumes volatility follows a random process negatively correlated with the evolution of the asset price as indicated by market data. This has been regarded as a valuable improvement over the Black-Scholes model and demonstrates the volatility smile phenomenon. The value of this improvement is evidenced further in the application of Heston's model to the Mexican financial market. It is important to note, however, that the Heston model remains far from perfect; most notably, its framework cannot adequately explain stock market crashes which result from large jumps in the magnitude of financial returns.

For a European call option with strike price K and maturity T , the Heston model gives the call value as:

$$C(S_0, K, T) = S_0 P_1 - K e^{-rT} P_2$$

where P_1 and P_2 are risk-neutral probabilities obtained from the Heston characteristic function.

The probabilities are calculated from:

$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} [e^{-iu \ln K} f_j(u) / (iu)] du$, $j = 1, 2$ An equivalent and commonly used representation is:

$$P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} [e^{-iu \ln K} \phi_1(u) / (iu)] du \quad P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} [e^{-iu \ln K} \phi_2(u) / (iu)] du$$

The Heston model is a widely used mathematical model in financial derivatives pricing, especially for options. It improves on simpler models (like Black-Scholes) by allowing volatility to change over time instead of being constant.

The Heston model is defined by two stochastic differential equations:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW^S$$

$$dv_t = \kappa(\theta - v_t) dt + \sigma \sqrt{v_t} dW^v$$

S_t : asset price (stock), μ : expected return, v_t : variance (volatility²), dW^S : random market shock, and volatility is not constant. v_t : variance at time t , κ : speed of mean reversion,

θ : long-term average variance, σ : volatility of volatility ("vol of vol"), dW^v : another random

shock.

Current variance (v): 1.04%, Long-term variance (θ): 0.04, Speed of reversion (κ): 2 Time step: 1

The Heston formula for $P_2 = 21 + \pi \int_0^\infty f_2(u) du$.

$\ln(S_0/K) = -0.05$ and $e^{-rT} = 0.998$, correlation coefficient = -0.5 $\sigma v = .35$ $k = 2.5$

$\sigma v = -0.175$ and $\rho \sigma v^2 = 0.1225$ $\kappa \theta / \sigma v^2 = 3.47$ $\varphi(-i, T) = E[ST]$.

$\varphi(-i, T) = 54958(1.001507985)$ $P_2 = 0.5 - 0.43162648 = 0.0683$ $P_1 = 0.07311$

C Heston = Rs 58.03

CONCLUSION:

Using the binomial model, when the strike price is 58,000, the theoretical call value is approximately ₹427.37. When the strike price is 60,000, the theoretical call value decreases to approximately ₹139.06. On Jun 29th, the actual call values are found to be Rs 153 and Rs 4 for strike prices of 58000 and 60000, respectively. The binomial model is useful in understanding variation of asset price with small changes by $u = d = 2\%$ and for $u = d = 1\%$, call price is Rs 36 for strike price 58000 and Rs 0.84 for strike date 60000. Using the Black-Scholes (B/S model) model call value can be derived. On 10th June, with $S = 54958$ for strike price of 58000, the BS model returns a call price of Rs 22 and a put price of Rs 2913, which differ significantly from the actual values of Rs 125 and Rs 4866 for the call and put options, respectively. Similarly for strike value of 60000 the call and Put values from the model differ significantly from the actual values. As the risk-free interest rate is approximate and volatility is not accurately predictable, the model values differ, and they are approximations. The Heston model for strike value 58000 yields Rs 58 which is also different from Rs 288 for observed date 10th June. In Heston model exact value of volatility of volatility is not determinable as data is for a month and also mean reversion values is also approximation. There lies further scope to use Reverse Fourier transform to analyse option pricing.

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Web sources:

a) <https://www.nseindia.com>

b) <https://www.nseindia.com/option-chain>

R code:

```
# Set parameters S0 <- 56813
```

```
mu <- 0.05
```

```
sigma <- 0.01077 dt <- 1/365
```

```
# Number of days n <- 11
```

```
# Set seed so that the simulation can be reproduced set.seed(123)
# Generate random standard normal values epsilon <- rnorm(n, mean = 0, sd = 1)
# Create vector for stock/index prices S <- numeric(n + 1)
S[1] <- S0
# Simulation for (t in 1:n) {

S[t + 1] <- S[t] +
mu * S[t] * dt +
sigma * S[t] * epsilon[t] * sqrt(dt)
}
#Display results simulation <- data.frame( Day = 0:n,
Epsilon = c(NA, epsilon), S = S
)
print(simulation)
# Price on 11th day S_11 <- S[12]
cat("Simulated price on Day 11 =", S_11)

And code for 10000 simulations and mean value prediction for asset price on 29 Jun # Parameters
S0 <- 56813
mu <- 0.05
sigma <- 0.01077 dt <- 1/365
# Number of simulations and days N <- 10000
n_days <- 11
# Set seed for reproducibility set.seed(123)
# Store final Day-11 prices S11 <- numeric(N)
# Monte Carlo simulation for (i in 1:N) {
S <- S0
for (t in 1:n_days) {
# Generate standard normal random number epsilon <- rnorm(1, mean = 0, sd = 1)
# Calculate change in S delta_S <- mu * S * dt +
sigma * S * epsilon * sqrt(dt) # Update S
S <- S + delta_S
}
# Store Day-11 price S11[i] <- S
}
```

```
2)# Summary statistics mean_S11 <- mean(S11) sd_S11 <- sd(S11) min_S11 <- min(S11) max_S11
<- max(S11)
# 95% prediction/interval range of simulated prices lower_95 <- quantile(S11, 0.025)
upper_95 <- quantile(S11, 0.975)
# 95% confidence interval for the estimated mean SE <- sd_S11 / sqrt(N)
CI_lower <- mean_S11 - 1.96 * SE CI_upper <- mean_S11 + 1.96 * SE # Display results
results<- data.frame( Statistic = c( "Mean",
"Standard Deviation", "Minimum", "Maximum",
),
Value = c( mean_S11, sd_S11, min_S11, max_S11, lower_95, upper_95, CI_lower, CI_upper
)
)
print(results)
```